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Architecture and Economics for Grid Operations 3.0

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Architecture and Economics for Grid Operations 3.0

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ABSTRACT

This monograph presents a possible research agenda for analytics and control of a deep decarbonized electric grid with pervasive data, interactive consumers, and power electronics interfaces. It focuses on new lines of investigation that are driven by new technological, economical, and policy factors. Conventional monitoring and control of the power grid heavily depends upon the physical principles of the underlying engineering systems. There is however increasing complexity of the physical models compounded by a lack of precise knowledge of their parameters, as well as new uncertainties arising from behavioral, economic, and environmental aspects. On the other hand there is increasing availability of sensory data in the engineering and economic operations and it becomes attractive to leverage such data to model, monitor, analyze, and potentially close control loops over data.

The increasing deployment of large numbers of Phasor Measurement Units (PMUs) provides the potential for providing timely and actionable information about the transmission system. Chapter 2 examines a framework for drastically reducing the dimensionality of the high volume streaming data, while preserving its salient features for purposes such as event detection, classification and visualization, and potentially even to close the loop around

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the data. Driven by the deepening penetration of renewable energy resources at both transmission and distribution levels, there is an increasing need for utilizing power electronics interfaces as intelligent devices to benefit the overall grid. Chapter 3 offers a conceptual design and concrete examples of a qualitatively different power grid stabilization mechanism in the context of networked microgrids. Another major paradigm change in the operation of the grid is that demand will have to be engaged much more to balance the partially variable renewable energy supply, which in turn requires greater understanding of human behavior to economic variables such as price. Chapter 4 presents a possible formulation to model the behavior of individual consumers in future grid operations. Chapter 5 presents a proposed solution to the problem of detecting attacks on the sensor measurements in the grid, which has become a greater concern with increasing reliance on sensor data transported over communication networks, with both sensors and networks liable to malicious cyber-attacks.

The goal of this monograph is to design clean, affordable, reliable, secure, and efficient electricity services. and to expand the horizon of the state of the research in the electric energy systems, at a critical time that is seeing the emergence of Grid 3.0. It is by no means complete and aims to stimulate research by next generation researchers.

1

Introduction

The electric utility industry designed, built, and operates the largest and most complex engineered system on the planet, and gave it a level of reliability that is unmatched by any other manufacturing industry (National Academy of Engineering, 2015). While constantly-evolving, it can be characterized roughly by three major milestones (NIST, 2015).

As the legacy grid of the 20th century,"Grid 1.0" can be thought of as interconnections of the bulk electric power system. Industrialized economies, and the developing nations, have gone through this process. The major technologies introduced during this generation set the foundation of the modern electric energy systems. The key features of electricity services in this generation can be summarized as follows: (1) the electricity generation centers are located far away from the load centers; (2) electricity is delivered from the generation centers to the load centers through alternating-current (AC) long-distance transmission systems; (3) the price of electricity is mostly determined by government policies, with no market competition involved.

In the past 10 to 15 years, "Grid 2.0" saw the emergence of industry deregulation and attempted to introduce market-based solutions to wholesale electric energy systems. Most industrialized economies have gone through this generation and adopted various electricity market policies. The price of electricity is determined by a competitive electricity market rather than fixed government policies. This could benefit consumers by lowering electricity prices, while offering the public more choices on the types of paid services.

"Grid 3.0" refers to what is happening now and is likely to proliferate in the next few decades: a smarter grid that integrates many more diverse resources and decision makers through a more flexible delivery system, with the overall objective of achieving cleaner, more affordable, and more reliable electricity services. The power grid is undergoing profound changes as a key enabler of sustainable societal and economic development in the 21st century. Fossil fuel based generation is being rapidly replaced by renewable energy resources. In the U.S., more than 16 GW of coal generation retired from the fleet in 2015 alone, while in the same year more than 9 GW of wind and solar power was integrated to the grid. A key research question during this paradigm change is that many objectives of the operation do not align well in a renewable-dominant power system. One example is that the objective of reducing greenhouse gas emissions by replacing conventional fossil fuel generation with renewables can have an adverse impact on grid reliability and efficiency. For instance, in California the high amount of photovoltaic (PV) penetration has led to the infamous "duck curve" where to maintain power balance there is a substantial need for fast ramping fossil fuel generation, which in turn reduces the environmental benefits of renewables.

In order to fully realize the premise of great individual pieces of technology (such as advanced power electronics control of PV panels and new sensory data), we are taking a path that will integrate the grid with (i) massive amount of streaming data; (ii) pervasive power electronics interfaces; and (iii) human-in-the-loop economics. We define this vision as "Grid 3.0". The research agenda is how to monitor and control a deep decarbonized grid with the above three driving forces.

1.1 Driving factors of the Grid 3.0 transition

A number of technological, economical, and policy drivers are behind the "Grid 3.0" transition, as illustrated in Figure 1.1.

On the *technological* front, much higher levels of modeling and operational uncertainties arise due to the drastic change of the generation portfolio. Some 200 GW of renewable variable energy capacity contributes to about 20%

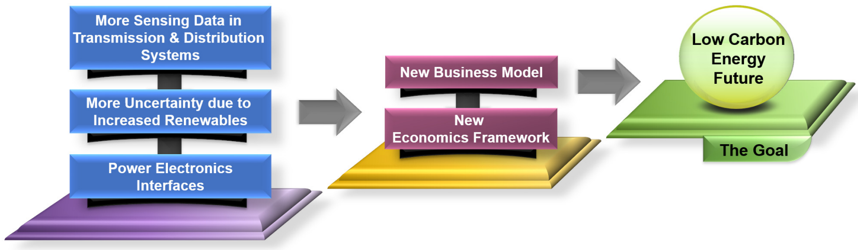

Figure 1.1: Driving factors of the Grid 3.0 transition.

of the entire generation capacity in the U.S., while 16 GW of coal generation retired in 2015 alone with more to come in the next couple of decades (US Energy Information Administration, 2017). The reliability and security [1] of the grid will need to be carefully engineered for such a change of generation portfolio. The conventional boundary between transmission and distribution is also rendered less clear due to the fact that many new resources are directly integrated at the distribution levels. For example, in the European Union, more than 90% of the newly installed solar PV is integrated at lower voltage distribution systems in 2016 (European Renewable Energy News, 2017). The increasing level of human-in-the-loop decision making from demand response strategies further compounds the modeling complexity and uncertainty. This potentially shifts the focus of research from primarily addressing optimization of the high voltage network to utilizing and controlling the distribution systems.

The above significant change in the generation resource mix also raises challenges to the resilience of the future electric grid. As more intermittent and uncontrollable renewable resources are being connected to the grid, and as extreme weather conditions happen more frequently, future electric grids are being threatened by grid resilience and fuel security issues. To resolve the resilience challenges, Federal Energy Regulatory Commission (FERC) issued an order in January 2018 (FERC, 2018) and sought for comments on better definition of grid resilience, methods for assessing and measuring resilience risks, as well as potential mitigation measures of resilience problems. Besides,

[1] Power system security is the ability to maintain the flow of electricity from the generators to the customers, especially under disturbed conditions.

for systems with deregulated electricity markets, the proper market mechanism is needed for addressing the grid resilience challenges.

Enabling technologies also can potentially lead to a fundamental rethinking of how to model, analyze, and control the grid. Over the past decade, billions of dollars have been invested in deploying tens of millions of sensors such as smart meters and thousands of PMUs in the electric grid. This large scale deployment has enabled the collection of a massive amount of data. The payback from this huge investment in data infrastructure is anticipated to be (a) more flexibility from demand response participation for smart aggregators; and (b) improved real-time situational awareness for the grid operators. The streaming data in the smart grid thereby provides unprecedented opportunities to transform the operation of the grid.

Another key technology enabler during the Grid 3.0 transition is potentially pervasive power electronics-based interfaces with renewable generations and price-responsive demand/loads. By leveraging highly controllable power electronics (PE) interfaces – voltage source inverters (VSIs), and advanced measurement technology – PMUs, novel control strategies can be applied, through which desirable power sharing behavior among coupled microgrids can be achieved. These PE interfaces can change the interface dynamics between microgrids and the bulk transmission grid in several ways:

- PE interfaces can change grid operations from simulation-based operations to design-based operations.
- PE interfaces can change distribution system loss minimization and reactive power management from a top-down centralized model-based approach to bottom-up adaptive approach.

On the *economics* front, a key driver is the potential collapse of the conventional commodity business model for the utilities. A prime need for utilities is to exploit new business models for electric distribution systems. Electricity sales by utilities in many areas are declining for a variety of reasons, including increased efficiency and conservation, and distributed generation. For utilities to remain viable and ensure continued reliability of power delivery, new business models that incentivize distribution systems to invest in capital purchases and operations are needed.

Coupled with technological and economic drivers, a third dimension of *policy* driver around the globe is the movement towards low carbon energy systems (Chu and Majumdar, 2012; United Nations, 2015). Driven by this agenda, many more renewable energy resources are being integrated into the system. Therefore, there is an increasing need for leveraging flexibility from the demand side to partially balance the intermittent supply.

1.2 Grid 3.0 research agenda overview

The objective of this monograph is to present the research problems in the new era of Grid 3.0, as well as possible research opportunities that could pave the way for such a transition. The key is to leverage the pervasive data, control, communication, and computation capabilities, that are becoming available at the end-user household interface, and to provide an *engineering and economics* model and solution for them to *provide, rather than only consume* a variety of services that are critical to the reliability of the grid operation. Figure 1.2 describes the authors' view of a possible research agenda.

Figure 1.2: Overview of a possible Grid 3.0 research agenda.

1.2.1 Closing the loop around data

Operation in the power grid involves with a large amount of streaming data. The proliferation of new sensors such as PMUs and smart meters provides much more data than conventional operation would have been able to manage. On the other hand, the increasing complexity of the grid requires the operators to make quicker and more adaptive decisions in near real-time. Data science methodologies including data fitting, data mining, machine learning, system identification, and adaptive control offer great opportunities in the new era of operating the grid. In this monograph, we focus on the following data science applications for energy system planning and operations:

- Data-driven situational awareness via dimensionality reduction.
- Data-driven low-quality data detection for PMU systems.
- Power plant model validation using PMU measurements.

In Chapter 2, the problems of utilizing streaming PMU data's low dimensionality and sparsity to conduct early anomaly detection, data quality filtering, and power plant model validation are described.

1.2.2 Modeling and control the grid with power electronics interfaces

Most distributed and renewable energy resources interact with the AC grid through power electronics interfaces. While many efforts have been devoted to this endeavor, the full potential of a provably reliable, "plug-and-play" distribution grid that supports cost-effective integration of sustainable resources has not yet been realized. Foreseeing that disruptive, low-cost sensing (e.g., micro-PMUs) and power electronics (e.g., voltage source inverters) technologies are on the horizon, we propose a clean slate approach to rethinking the control architecture of the distribution grid. In sharp contrast with today's paradigm in which the distribution grid serves as a passive, one-way, tree-structured energy delivery system to end users, we envision a future distribution grid as an open-access, active platform that supports multiple coupled and operated microgrids. In short, each microgrid can serve as the intelligent periphery of a smart distribution grid of the future (Bakken *et al.*, 2011). In Chapter 3, we describe such a possible framework with voltage angle droop control of power electronic interfaces for guaranteed dynamic stability in microgrid interconnections.

1.2.3 Grid interaction with human-in-the-loop

The operation of electric power systems has traditionally adopted the philosophy of controlling generation to match the stochastic demand. As a result, dynamic modeling and control of power systems has been primarily focused on the generator side. Governor-turbine-generator (GTG) modules for various fossil fuels are modeled from first principles, resulting in a mature modeling taxonomy with well engrained notions such as droop characteristics and ramp rates. More recently, there has also been work done on the modeling of renewable energy sources. Wind farms have been modeled as stochastic dynamic systems controlled by doubly-fed induction generators. During the era where the prevailing paradigm was that supply follows demand, this modeling of the supply side was sufficient to develop a coherent resource allocation framework for electric power systems. However, with the advent of demand response where demand too can be viewed as a controllable entity, it has become imperative to symmetrically develop models for analyzing demand response too as a dynamical system with well defined inputs and outputs.

The central challenge of modeling demand response is that human behavior is involved in the decision making process. In Chapter 4 we outline a possible framework at both *wholesale* and *retail* levels to modeling grid interactions with energy consumers. At the wholesale level, we present a dynamical systems perspective to modeling price responsive demand. At the retail level, we present an empirical exercise in engaging end users in coupon incentive-based demand response.

1.2.4 Detecting cyber-attacks on sensor measurements

The introduction of more sensors to monitor the grid state, and the increasing utilization of networks to transport the sensed data, permit better operation of the grid. However, at the same time, they also increase the vulnerability of the grid to cyber-attacks on sensors and networks. Indeed this has become a major concern after reports of several attacks on industrial control systems and infrastructure.

In Chapter 5, we present a method for detecting cyber-attacks that is based on active injections of "watermarks" into the grid, and testing whether information flows carry the right transformations of these watermarks. This provides a general purpose method for detecting attacks in "cyber-physical systems." We provide an analysis of this method and report on how it performs in a simulation study in defending against attacks on Automatic Generation Control.

1.3 Monograph organization

The rest of this monograph is organized as follows: Chapter 2 discusses the possible ecosystem for closing the loop around data-driven technologies in electric energy system planning and operations; Chapter 3 proposes the possible power electronics applications for enhancing the end user experience. Chapter 4 presents a future modeling framework for improving demand-side economic efficiency. Chapter 5 presents an active method for detecting cyber-attacks on sensor information in the grid. Chapter 6 provides concluding remarks.

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